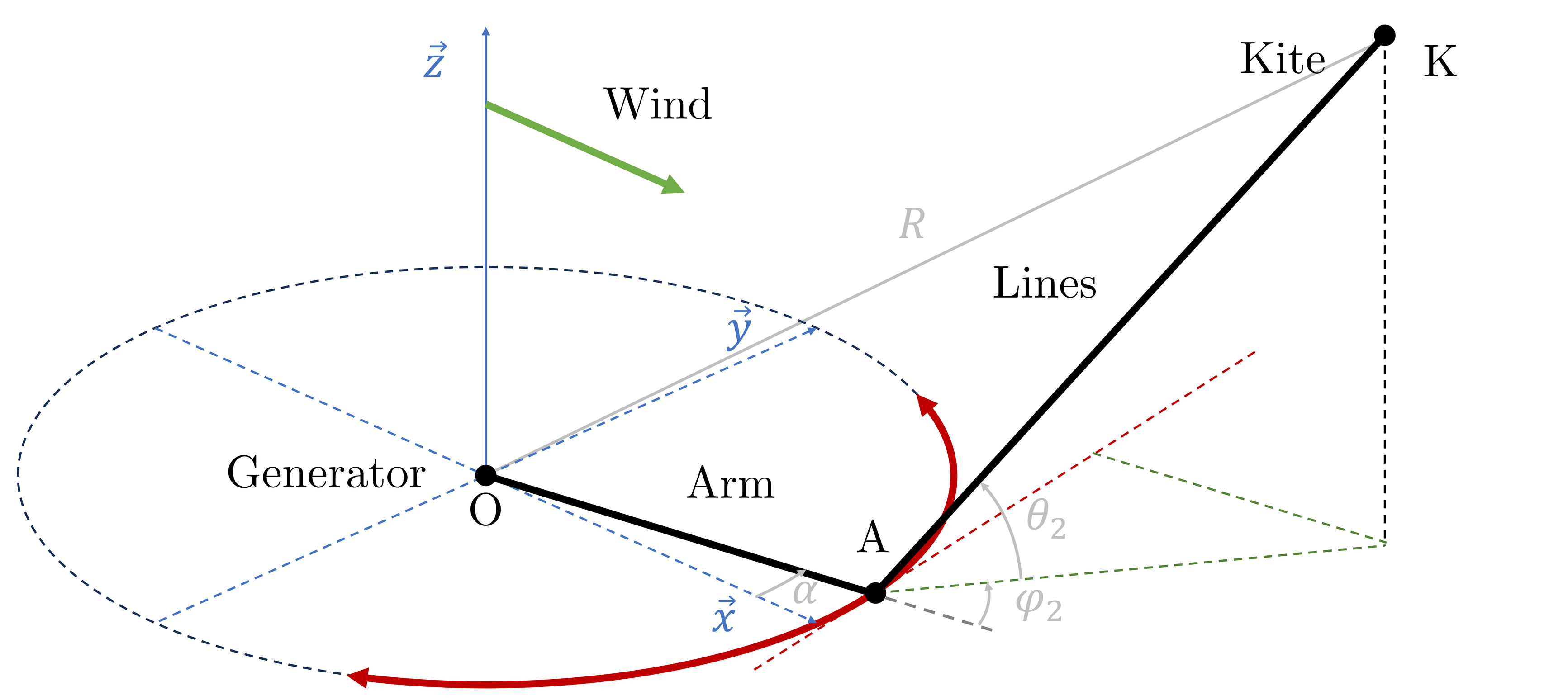
Kite Electrical Energy Production

Antonin Bavoil[1], Kevin Desenclos[2]
[1] Université Côte d'Azur, CNRS, Inria, LJAD [2] ENSTA Bretagne

Energy harvesting using kites

Our collaborators from Brest have previously worked with French navigator Yves Parlier on ship propulsion using kites. Following this project, they teamed up with INRIA to explore terrestrial electricity generation using an original design.



Modelisation of the device

The lines are considered to be one solid segment while the wind only depends on the altitude. We consider the dynamic of the kite's center of gravity and the arm's orientation, which is computed using three forces :

- weight of the kite and the lines

$$\vec{F}_{\text{gravity}} = \left(m_{\text{kite}} + \frac{m_{\text{lines}}}{2}\right)\vec{g}$$

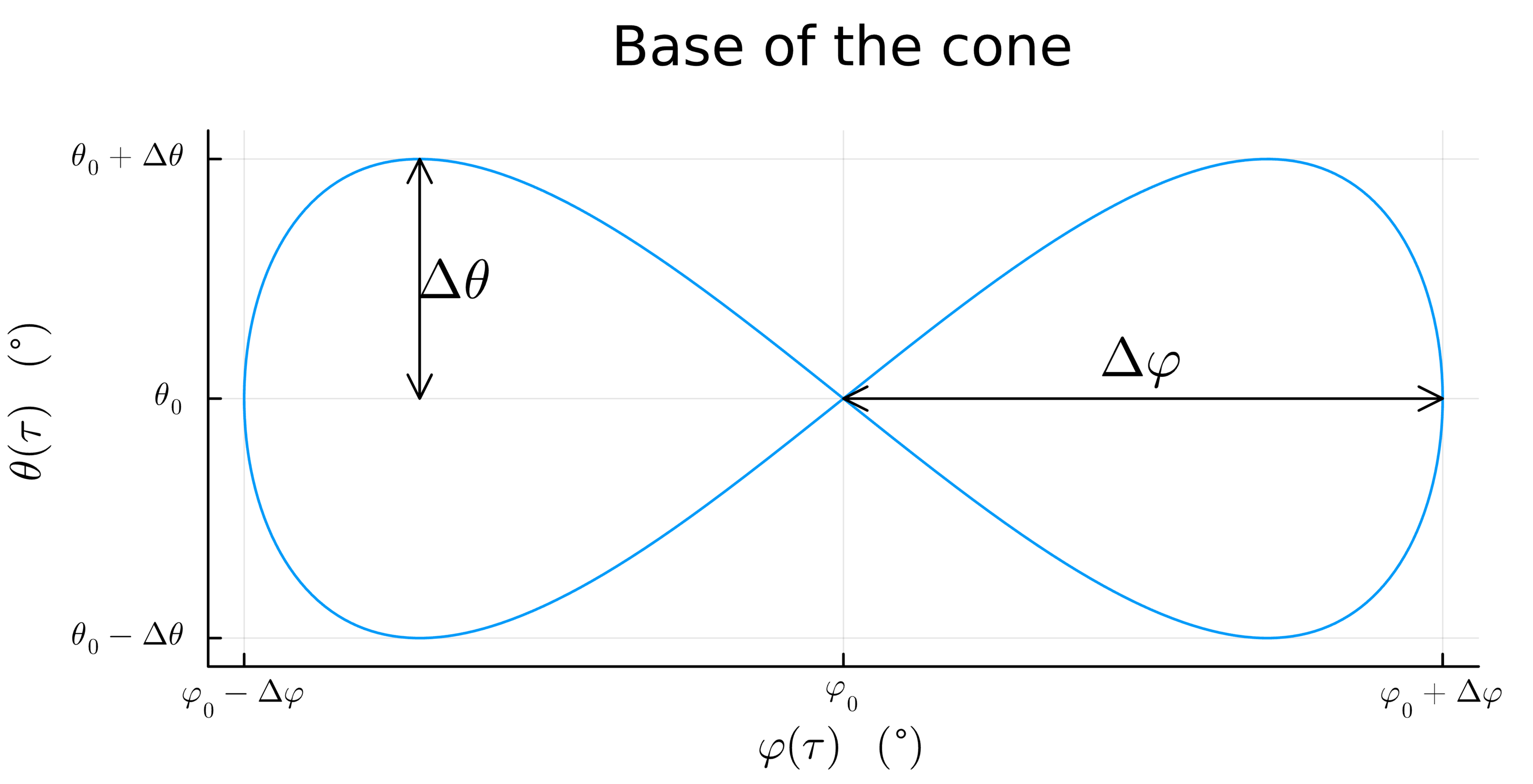
- aerodynamical forces on the kite and on the lines, where $\vec{w}_{app}$ is the apparent wind

$$\vec{F}_{\text{aero}} = -\frac{1}{2} S \rho_{\text{air}} \left\|\vec{w}_{\text{app}}\right\|^2 (C_L \hat{z}_w + C_D \hat{x}_w) - \frac{n b_l \rho_l r d_l}{6} \left\|\vec{w}_{\text{app}} - (\hat{r}\cdot\vec{v})\hat{r}\right\|^2 C_{Dl}\, \hat{e}_w$$

- line tension on the kite

$$\vec{F}_{\text{tension}} = \frac{I_{\text{eq}}\ddot{\alpha} - C_{\text{gen}}(\dot{\alpha})}{l \sin(\theta_2)\sin(\varphi_2)}\hat{r}.$$

At this point, we want to keep a minimalistic model, hence we replace the control of the kite by a geometrical constraint: it should stay on an 8-shaped cone. Let us add a fictitious force $\vec{F}_{\text{cone}}$ that acts as a control so that the kite glides frictionless on this cone; $\vec{F}_{\text{cone}}$ should be othogonal to the cone at all times. This cone is parametrized by R the distance to the origin and $\tau \in [0, 2\pi]$ the parameter of the eight.



Definition of the problem

As our variables, we choose the three angles α, θ_2 and φ_2, the parameter of the cone τ, and the distance from the kite to the origin R. Hence, we have a state q composed of five variables and their time derivatives. The force $\vec{F}_{\text{cone}}$ being orthogonal to the cone gives two equations:

$$\vec{F}_{\text{cone}} \cdot \frac{\partial \overrightarrow{OK}}{\partial R} = 0 \text{ and } \vec{F}_{\text{cone}} \cdot \frac{\partial \overrightarrow{OK}}{\partial \tau} = 0$$

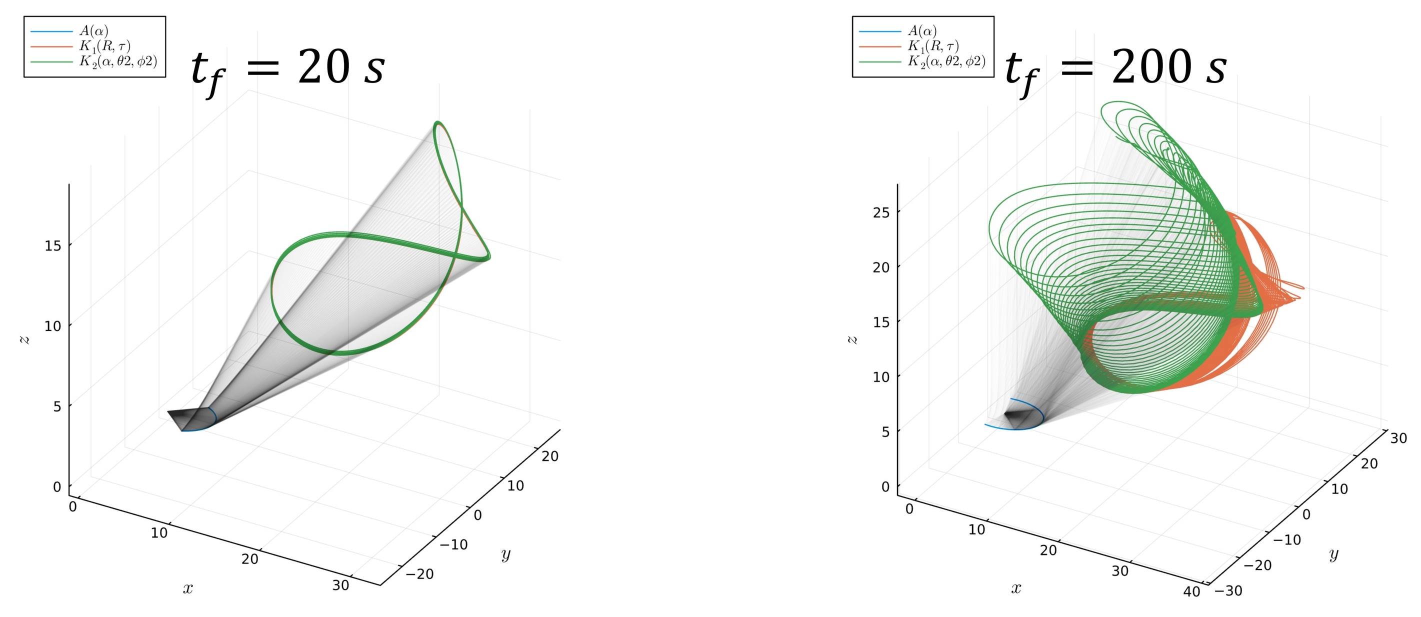
where $\vec{F}_{\text{cone}} = m\,\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} - \vec{F}_{\text{gravity}} - \vec{F}_{\text{aero}} - \vec{F}_{\text{tension}}$.

The position of the kite is over-described, hence we enforce that the state remains consistent with the three following equations:

$$\frac{\mathrm{d}^2 \overrightarrow{OK}(R,\tau) - \overrightarrow{OK}(\alpha,\theta_2,\varphi_2)}{\mathrm{d}t^2} = 0_{\mathbb{R}^3}.$$

Resolution of the ODE

We set the initial state as a plausible value, and let Runge-Kutta 4-5 run for twenty and two-hundred simulated seconds respectively.

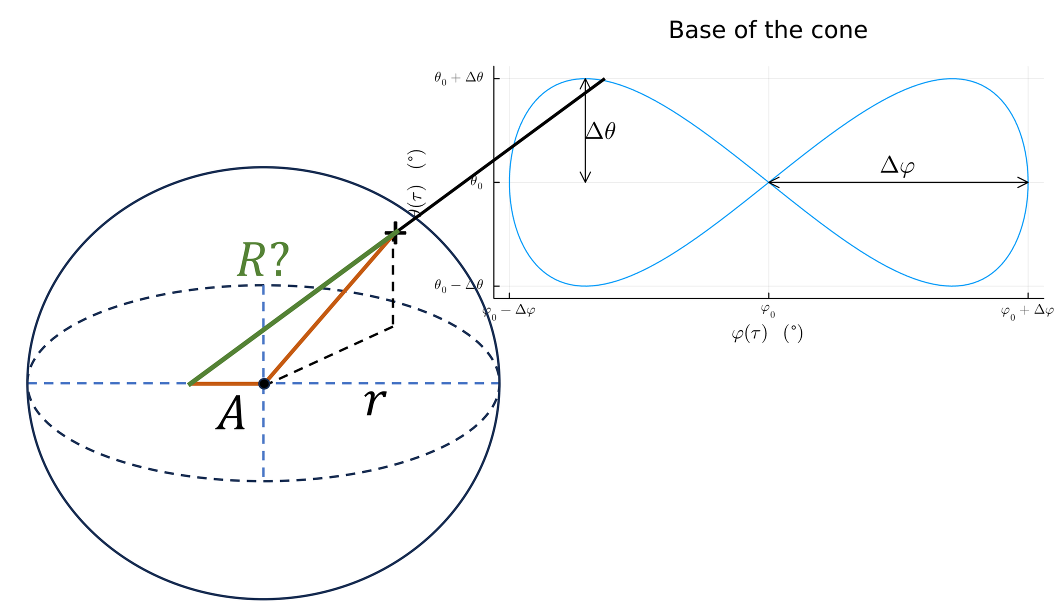


Callback

Because of the over-description of the kite's position, the simulation becomes inaccurate for longer simulation durations. To remedy this, we define the following residual function:

$$g(q,\dot{q}) = \begin{pmatrix} \overrightarrow{\mathrm{OK}}(R,\tau) - \overrightarrow{\mathrm{OK}}(\alpha,\theta_2,\varphi_2) \\ \frac{\mathrm{d}\overrightarrow{\mathrm{OK}}(R,\tau) - \overrightarrow{\mathrm{OK}}(\alpha,\theta_2,\varphi_2)}{\mathrm{d}t} \end{pmatrix} \in \mathbb{R}^6,$$

and at each timestep, we project our solution on the manifold $g = 0$. This idea works, but we might aswell get rid of the over-description by having an adequate state. Indeed, α and τ fully describe our system; in particular, with some geometry we can reconstruct R and $\vec{F}_{\text{tension}}$.



Optimisation problem

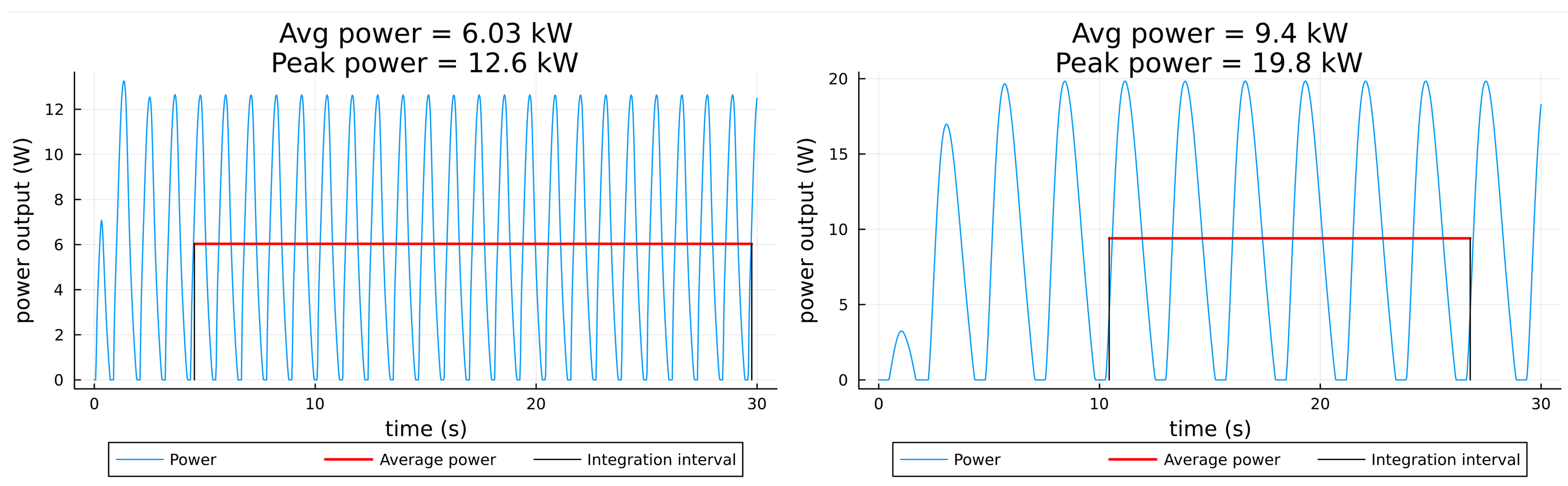
Leveraging the reduced model in dimension four of the problem, we eventually want to find the optimal parameters (length of the arm, length of the lines, inertia of the a...) that would produce the most average power. We define the following gain function, that we seek to maximize:

$$\int_{t_0}^{t_f} P(t)\,\mathrm{d}t = \int_{t_0}^{t_f} \dot{\alpha}\, C_{\text{gen}}(\dot{\alpha})\,\mathrm{d}t \to \max.$$

We set box constraints on the parameters and use Particle Swarm Optimization to solve the problem.

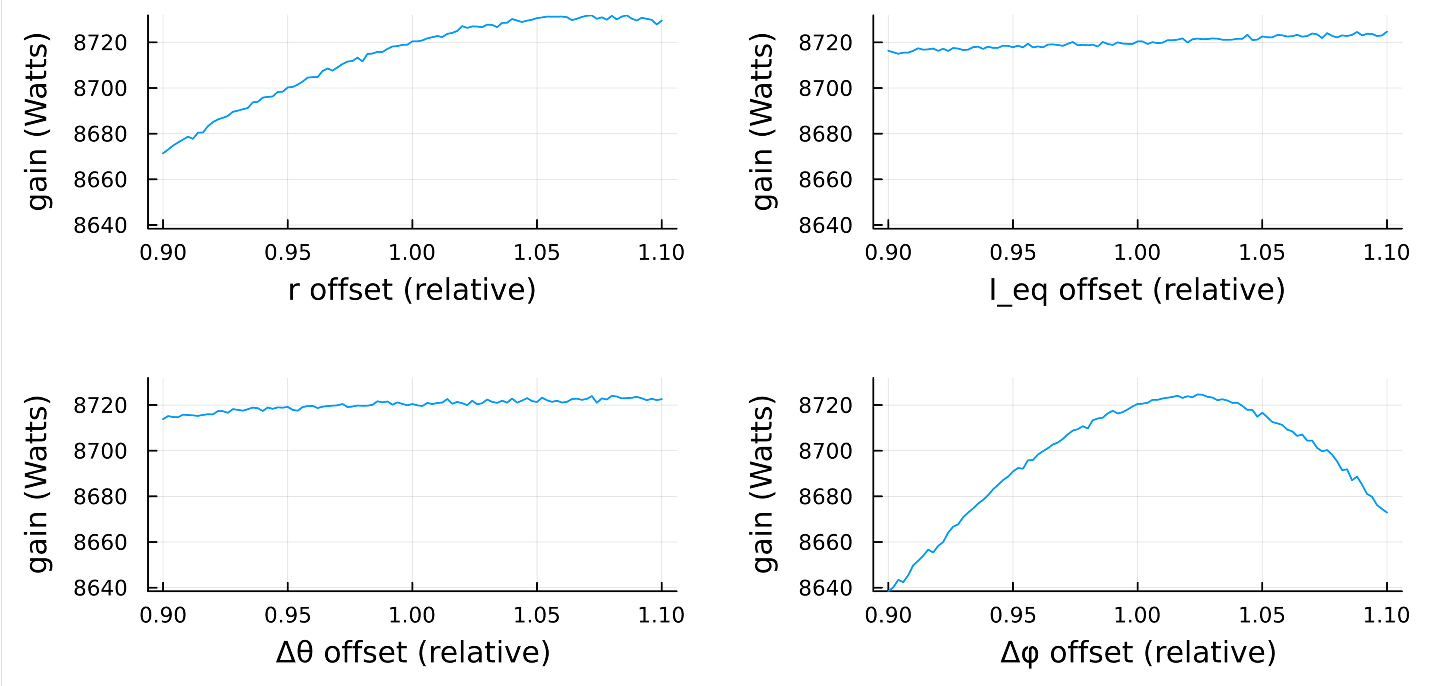
Initial parameters

Optimised parameters



Landscape around the optimal point

Knowing the optimal parameters is a nice information, but knowing the sensibility of the gain function with respect to each parameter is much richer. This last plot shows the landscape of the gain function around the optimal point. An ongoing work is to restrict our optimisation to the limit cycle only.



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